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*Manuscript received April 19, and accepted April 29, 1983.*


# Direct Contact Heat Transfer with Phase Change: Motion of Evaporating Droplets


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Direct contact heat transfer between liquids has the advantage of eliminating metallic heat transfer surfaces which are prone to corrosion and fouling. In addition, if phase change occurs, a larger heat capacity for heat absorption is available. The mechanism of heat transfer between two immiscible phases and dynamics of a vaporizing drop are, relatively, more complex than either that of a drop or a bubble of constant radius.

As reported by Sideman and Taital (1964), Simpson et al. (1974), and Sambi (1981), a dispersed liquid droplet changes its shape from spherical through ellipsoidal to a cap-shaped bubble during the course of evaporation through continuous immiscible liquid medium. In addition to these changes in shape, the two-phase bubble oscillates causing the unevaporated dispersed liquid in the two-phase bubble to slosh from side to side. The combined effect of irregular transformation in the shape of the two-phase bubble, sloshing of the unevaporated liquid in the two-phase bubble, and its zigzag trajectory has complicated the study of the mechanism of heat transfer and dynamics of a vaporizing drop.

As far as theoretical expressions for heat transfer involved, motion of the evaporating two-phase bubble, and the total time of its evaporation are concerned, the complicated nature of the problem has stood in the way of workers in developing suitable models that could represent the experimental data.

In pursuit of such a study, a mathematical model for the heat transfer coefficient has already been developed (Raina and Grover, 1982).

This paper deals with the motion of the vaporizing two-phase bubble.

Based on the results of their experimental studies of the motion of expanding bubbles, Sideman and Taitel (1964) developed the following empirical formula for the pentane-water system describing the relationship between the position of the bubble and time.

$$H = H_0 + Bt^p$$

or

$$U = \frac{dH}{dt} = Bpt^p \tag{1}$$

Quantitative description of the behavior of single bubble and drop dispersions in continuous fluid is often based on the description of a free motion of a single bubble or drop whose shape is usually defined by $D$ of the volume equivalent sphere, that is

$$D = \left(\frac{6V}{\pi}\right)^{1/3} \tag{2}$$

The basic equation for the terminal velocity of a single drop is derived from the force balance equation

$$\pi D^3(\rho_c - \rho_d)\frac{g}{6} = C_D\left(\frac{\pi D^2}{4}\right)\left(\rho_c \frac{U^2}{2}\right)$$

or

$$U = \left[\frac{4}{3}\left(\frac{\rho_c - \rho_d}{\rho_d}\right)\frac{D{\cdot}g}{C_D}\right]^{1/2} \tag{3}$$

For an evaporating drop in another immiscible liquid, the expression can be written as

$$U = \left[\frac{4}{3}\left(\frac{\rho_c - \rho_{dav}}{\rho_d}\right)\frac{D{\cdot}g}{C_D}\right]^{1/2} \tag{4}$$

Investigation of energy equations using the experimental data of Sideman and Taitel (1964) and Sambi (1981) revealed that the heat transfer coefficient is higher for drops of small initial diameter, Figure 1. It was thought worthwhile to put the above expression in a form that would include initial diameter of the drop.

$$\frac{\rho_c}{\rho_{dav}} = \frac{\rho_c}{\left[\frac{\pi}{6}D_0^3\rho_d \Big/ \frac{\pi}{6}D^3\right]} = \frac{\rho_c}{\rho_d}\left(\frac{D}{D_0}\right)^3$$

or

$$\left(\frac{\rho_c - \rho_{dav}}{\rho_c}\right) = 1 - \frac{\rho_{dav}}{\rho_c} = 1 - \frac{\rho_d}{\rho_c}\left(\frac{D_0}{D}\right)^3 \tag{5}$$

Substituting Eq. 5 in Eq. 4 we obtain

$$U = \left[\frac{4}{3}\left\{1 - \frac{\rho_d}{\rho_c}\left(\frac{D_0}{D}\right)^3\right\}\left\{\frac{D{\cdot}g}{C_D}\right\}\right]^{1/2} \tag{6}$$

For bubbles ($R \geq 0.7$ mm) whose shape is nearly spherical, it is most convenient to determine the coefficient of resistance from the relation obtained by Levich (1949, 1952) that is

$$C_D = \frac{2}{3}\left(\frac{\rho_c g D^2}{1.82\sigma_c}\right) \tag{7}$$

where $\sigma_c$ is the surface tension at the bubble surrounding liquid boundary. Selecki and Gradon (1976) have used this relation in their theoretical expression for the motion of the two-phase bubble.


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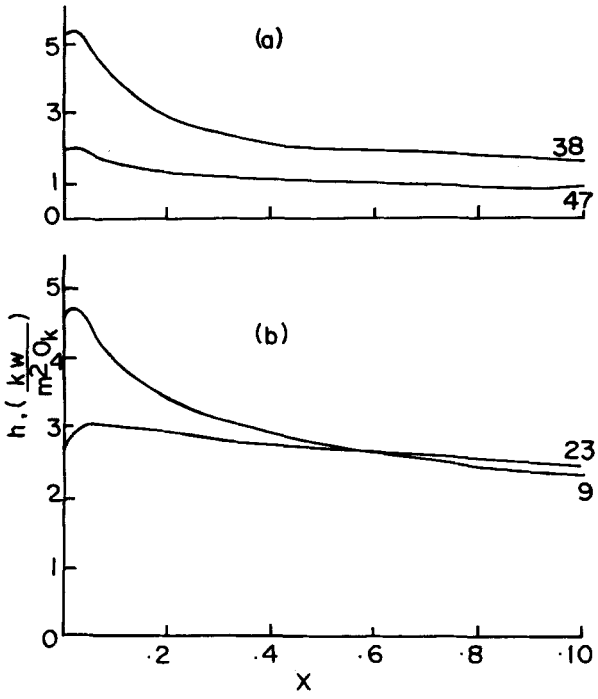



| FIG- 1-(a) | Run no | 38 | 47 |
|---|---|---|---|
| | $D_0$ | 1·10 | 3·50 |
| | ΔT | 6·8 | 6·8 |

| FIG- 1-(b) | Run no | 9 | 23 |
|---|---|---|---|
| | $D_0$ | 1·25 | 3·25 |
| | ΔT | 2·0 | 2·0 |

**Figure 1 (a) and (b). Instantaneous heat transfer coefficient pentane-distilled water system (Sambi 1981).**

Substituting the value of $C_D$ from Eq. 7 in Eq. 6 we obtain

$$U = 1.91\left[\left\{1 - \frac{\rho_d}{\rho_c}\left(\frac{D_0}{D}\right)^3\right\}\left\{\frac{\sigma_c}{\rho_c D}\right\}\right]^{1/2} \tag{8}$$

The value of instantaneous velocity predicted by Eq. 8 was within a tolerable limit (when compared with the experimental data of Sambi, 1981) during the early period of evaporation of the drop. As the size of the two-phase bubble increased, the predicted values showed a decreasing trend as against increasing values of velocity obtained experimentally. The authors found that the deviation between the predicted and the experimental values of velocity increased as the initial diameter of single, dispersed liquid drops increased. To account for the imbalance between the initial drop diameters and the initial diameter and instantaneous diameter, we introduced a dimensionless group, comprising initial diameter and instantaneous diameter in Eq. 8.

$$U = 1.91\left[\left\{1 - \frac{\rho_d}{\rho_c}\left(\frac{D_0}{D}\right)^3\right\}\left\{\frac{\sigma_c}{\rho_c D}\right\}\right]^{1/2}\left[\frac{1}{2}\left(\frac{D_0}{D} + \frac{D}{D_0}\right)\right]^{y} \tag{9}$$

where

$$y = \left[1 - \frac{1}{T_\infty}\left\{\frac{1}{2}\left(\frac{D_0}{D} + \frac{D}{D_0}\right)\right\}\right]$$

The experimental values of heat transfer coefficient for a given initial diameter of the dispersed liquid drop were found to decrease with increase in the temperature driving force. It was thought

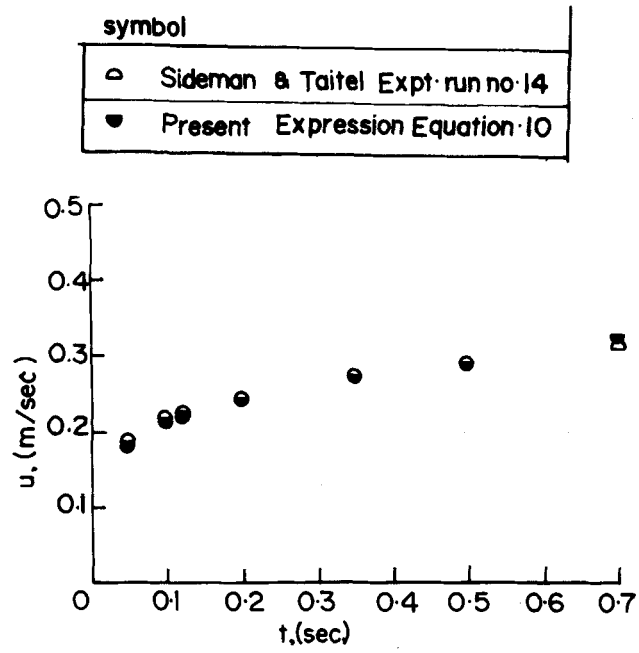



**Figure 2. Comparison of experimental and theoretical instantaneous velocity.**

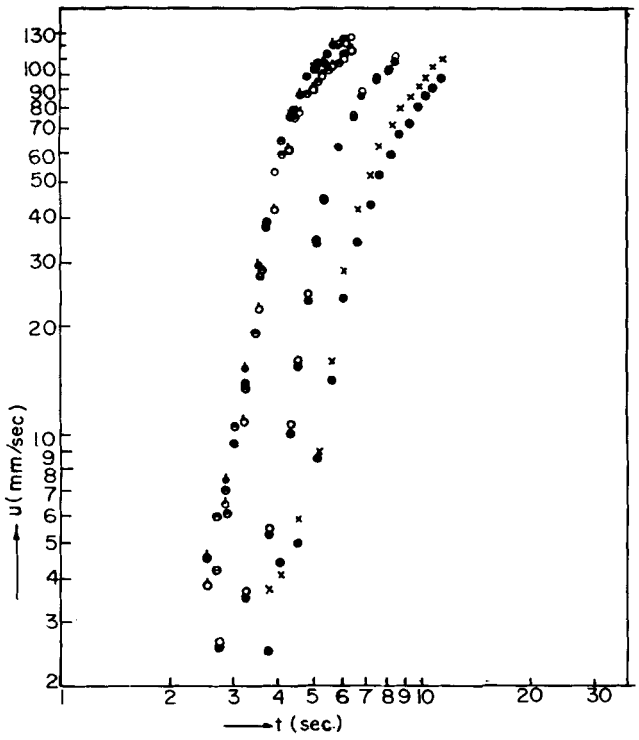
worthwhile to incorporate this effect also in a dimensionless form. The importance of introducing these variables in dimensionless form has already been displayed in a paper concerning diffusion (Raina, 1980), where the imbalance in the molecular weights of the solute and solvent (responsible for marked deviation from experimental values) is remarkably accounted for. A semitheoretical expression which predicts the values of instantaneous velocity very well is as under:

$$U = \frac{1.91\left[\left\{1 - \frac{\rho_d}{\rho_c}\left(\frac{D_0}{D}\right)^3\right\}\left\{\frac{\sigma_c}{\rho_c D}\right\}\right]^{1/2}\left[\frac{1}{2}\left(\frac{D^2 + D_0^2}{D \cdot D_0}\right)\right]^{y}}{\left[\frac{T_\infty^2 + T_L^2}{2T_\infty T_L}\right]^{[D^2 + D_0^2/2D_0 \cdot D]}} \tag{10}$$

Equation 10 is in good agreement with the experimental data of Sambi (1981) and Sideman and Taitel (1964), for systems involving the evaporation of dispersed liquid drops through less viscous liquid medium, and also serves very well (Figure 2) as a substitute to the graphical method (Eq. 1) adopted by Sideman and Taitel (1964). The expression holds well for all dispersed-phase drops evaporating through less viscous continuous-phase medium, covering drop size ranging from 1 to 4.5 mm initial diameter and the temperature difference between the two immiscible liquids from 1 to 14.1°C. However, it failed to predict the experimental data of Sambi (1981) and Tochitani et al. (unpublished) for systems involving high-viscosity continuous-phase liquid medium.

The experimental studies of Tochitani et al. (unpublished) and Sambi (1981) revealed that the velocity of a two-phase bubble in a high-viscous continuous-phase liquid is low compared to that of a less viscous continuous-phase liquid, and the two-phase bubble remains spherical and changes to a spherical-cap-shaped bubble towards the latter stages of evaporation, indicating that the viscosity of the continuous phase is a dominating parameter. We thought it worthwhile to introduce the effect of continuous-phase viscosity in the dimensionless form. Thus Eq. 10 was modified by introducing Prandtl number based on the properties of continuous-phase liquid on similar grounds as done by Isenberg and Sideman (1970). The expression which was found to be in reasonable agreement with the experimental data of Sambi (1981) for the viscous systems is as under:

$$U = \frac{1.91\left[\left\{1 - \frac{\rho_d}{\rho_c}\left(\frac{D_0}{D}\right)^3\right\}\left\{\frac{\sigma_c}{\rho_c D}\right\}\right]^{1/2}\left[\frac{D^2 + D_0^2}{2D \cdot D_0}\right]^{[5/6 - 1/T_\infty(D^2 + D_0^2/2D \cdot D_0)]}}{y' \cdot y''} \tag{11}$$




| Tochitani etal expti. run no. | $D_0$ (mm) | $\Delta T_\infty$ (°k) | SYMBOL expti. | SYMBOL predicted equation(11) |
|---|---|---|---|---|
| P-56 | 1·36 | 1·6 | ⊕ | x |
| P-67 | 1·37 | 6·1 | ⊖ | ⊖ |
| P-68 | 1·22 | 6·1 | ◇ | ◆ |
| P-108 | 1·42 | 3·1 | ○ | ● |

**Figure 3. Velocity of the two phase bubble vs. time.**

where

$$y' = \left[\frac{T_\infty^2 + T_L^2}{2T_\infty \cdot T_L}\right]^{[D^2 + D_0^2/2D \cdot D_0]}$$

and

$$y'' = \left[\frac{C_{p_c}\mu_c}{K_c}\right]^{D_0/1.6D}$$

It is clear from Figure 3 that Eq. 11 gives a satisfactory agreement with the experimental data (received from the authors in response to a request) of Tochitani et al. (1977), for small drops of less than 2 mm initial diameter of n-pentane evaporating through 98.3% aqueous glycerol.

As already reported (Raina and Gover, 1982) the mass balance equation in terms of the weight percent vapor content $x$ for the two-phase bubble can be written as

$$\left(\frac{D}{D_0}\right)^3 = 1 + x(m-1) \tag{12}$$

This expression enables us to get the instantaneous values of $D$ corresponding to various values of $x$.

Equation 10 can be safely used for all dispersed phases and low-viscous continuous-phase liquid systems. The systems which have been tried are: a) n-pentane, b) isopentane, c) n-butane, and d) furan as dispersed phases and distilled water and sea water as continuous phases. Equation 11 can be used for all dispersed phases cited above and aqueous glycerol as continuous phase.

## NOTATION

$B$ = constant
$C_D$ = drag coefficient
$D$ = equivalent spherical diameter of two-phase bubble
$g$ = acceleration due to gravity
$H$ = position of the two-phase bubble at any time during evaporation
$K$ = thermal conductivity
$m$ = ratio of liquid to vapor density of the bubble
$p$ = constant
$P_r$ = Prandtl Number, $\left[\frac{C_p\mu}{K}\right]$
$T$ = temperature
$t$ = time
$U$ = velocity of rise of two-phase bubble
$x$ = weight percent vapor

### Greek Letters

$\mu$ = viscosity
$\rho$ = density
$\sigma$ = surface tension

### Subscripts

av = average
$c$ = continuous phase
$d$ = dispersed phase
$L$ = between continuous and dispersed interface
$o$ = initial
$\infty$ = bulk

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*Manuscript received November 8, 1982; revision received March 8, and accepted March 16, 1983.*